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Various aspects of the Eulerian and Lagrangian predictability of oceanographically relevant flows are examined using models and data. For the problem of Lagrangian predictability, such as in search-and-rescue operations, dynamical systems techniques are used to determine the distributions of chaotic versus regular behavior in models of jets and recirculations. Eulerian predictability is addressed using phase space reconstruction of time series from models and hierarchy of oceanographic data sets. Uncertainty in flows in regions of complex, unresolvable topographic variations is examined using a porous medium approach.

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## **Development and Application of Dynamical Systems Techniques for Use with Ocean Data**

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### **Long Term Goals**

Long-term goals include improvement of our predictive capabilities and scientific knowledge concerning Eulerian and Lagrangian prediction in the ocean. Specific applications include search and rescue operations, the design of float and drifter experiments, the understanding of stirring mechanisms, and the quantification prediction errors associated with unresolvable complex topography. We are also interested in determining the extent to which underlying dynamics and predictability can be assessed solely on the basis of observed time series.

### **Objectives**

- (1) To determine the distribution of regions of Lagrangian chaos (as opposed to regular motion) in meandering, baroclinic jets;
- (2) To determine the usefulness of phase space reconstruction in assessing the dynamics and predictability of velocity time series from the Middle Atlantic Bight (MAB); and
- (3) To develop an approach for parameterizing the effects of sub-grid complex topography such as the Mid-Atlantic Ridge (Figure 1) or Aegean Sea archipelagos and assessing limits on predictability imposed by unresolved topographic features.

### **Approach**

Studies of meandering jets were carried out using a hierarchy of models, ranging from a simple analytical model of 2D flow to a 2-1/2 layer, quasigeostrophic model. The latter configured to roughly match the overall characteristics of the Gulf Stream. Various dynamical systems analyses, including the construction of invariant manifolds, were used to map out chaotic regions.

In the time series analysis of the MAB, we first applied phase space reconstruction to a hierarchy of data sets including data generated from the Lorenz model, a simple shallow water model of an ocean basin circulation, and tidal data taken from the MAB shelf area. We then applied the analysis to current meter records from the MAB shelf break area.

In the study of complex topography, a porous medium approach based on the 'island rule' was used to formulate a theory for circulation around the topography. The 'rule' was applied to a finite number of bumps or islands (Figure 2) and the dimensions of the bumps were then shrunk to infinitesimal values in order to approach the porous medium limit.

## **Tasks Completed**

Simulations and Lagrangian analysis of a number of oceanographic relevant flows, including jets and recirculations, were carried out. Phase space reconstruction was applied to a hierarchy of model and oceanographic data sets in order to determine the utility of the method. Analytical models of flow around complex topography were developed and tested against a barotropic model.

## **Results**

The studies of meandering jets (Yuan, et al., 2004a, and Yuan et al., 2002) have shown that Lagrangian chaos and cross-stream transport occur preferentially in the subsurface flow, probably 600-1200m in the Gulf Stream. This result is particularly satisfying as it was conjectured in earlier ONR-supported work (e.g. Pratt et al., 1995; Miller et al., 1997; Rogerson et al., 1999; Yuan et al., 2002), but never demonstrated. We are now beginning to develop a three-dimensional picture of chaos in these baroclinic flows.

The phase space reconstruction (Yuan, et al. 2004b) has yielded information about the simpler systems (the shallow water basin model and the tidal data) in terms of underlying nonlinearity. However, the method has proved less helpful when applied to the MAB data due to limitations in record length. However, the methods do suggest that predictability on the MAB is lost after a time period of only 12 hours.

The model of complex topography has yielded some surprising results concerning the porous nature of ridges and archipelagos (Pratt and Spall, 2003). It turns out that finer spacing between the bumps and islands can actually enhance the throughflow in some cases. The theory also provides a parameterization of complex topography that could be used in Navy models, including tidal models. Limits on predictability are currently being evaluated by stochastically forcing the differential equation that determines the throughflow.

## **Impact for Science**

The series of studies on Lagrangian chaos and predictability carried out as part of this DRI (see Rogerson et al. 1999; Miller et al. 2002; Deese et al. 2002; Yuan et al. 2002) has made a strong case for the relevance of chaotic advection as an important process in ocean stirring and Lagrangian predictability. (Previous attention had largely been focused on the atmosphere.) The phase space reconstruction work has provided advances in methodology and a better understanding on the advantages and limits of this technique. Limits on the predictability of complex regions such as the MAB are striking. The porous medium approach to complex topography has provided a fresh look and a new tool to be used in evaluation and parameterization of the effects of topographic features that are too fine to be resolved by standard prediction models of ocean circulation and tides.

## Relationships to Other Programs

Some of the Pratt and Spall (2003) work involves strait dynamics and overlaps with an ONR-supported study “Determining the stratification of exchange flows in sea straits.” This work also has a small overlap with NSF funding on deep overflows and has been acknowledged as such.

## Figures

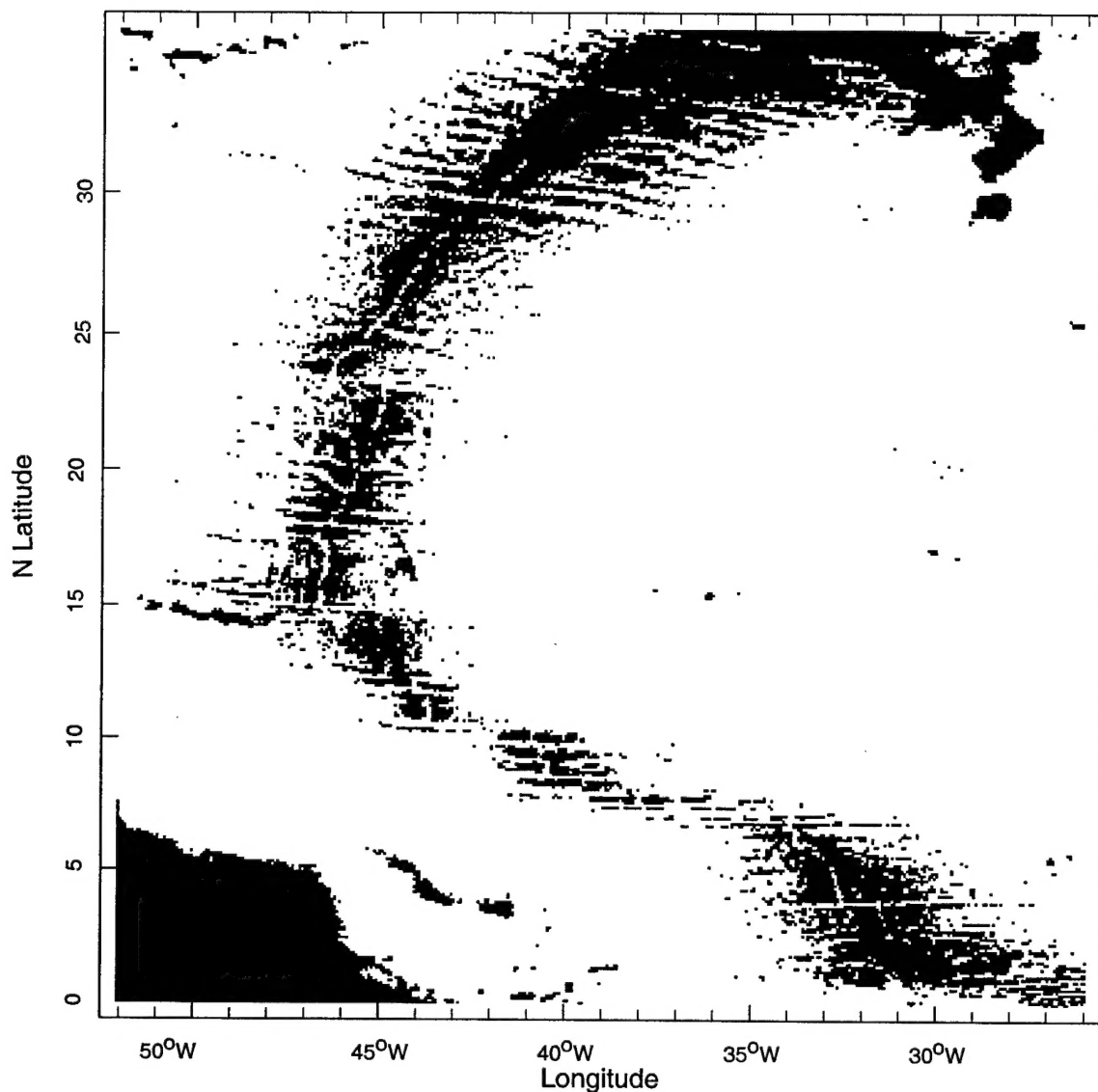


Figure 1. Slice through the Mid-Atlantic Ridge at 3500 m depth. This picture is typical of complex topography detail that is difficult to resolve in circulation and tidal models. Other areas of complexity include the ridges of the Indian Ocean, the Indonesian Seas, and the Aegean Sea.

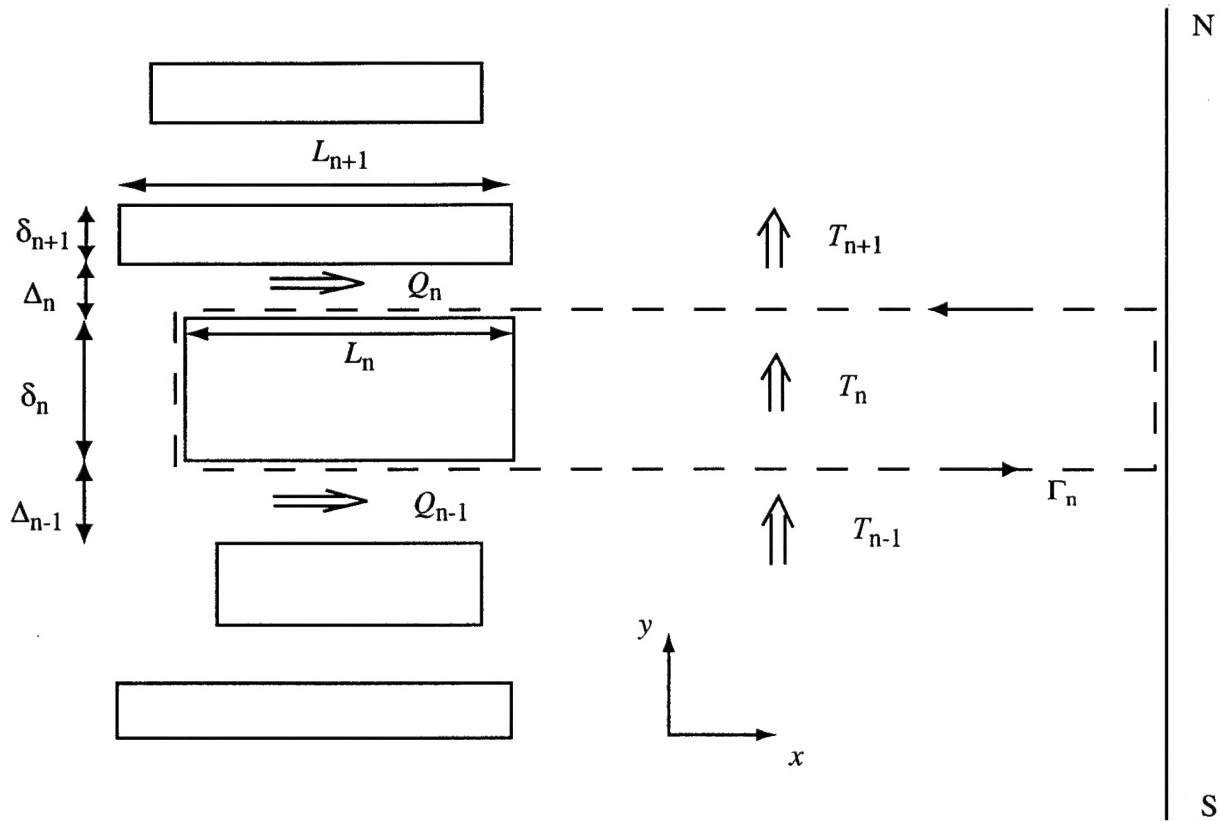


Figure 2. Islands such as those shown here are pieced together to form porous ridges. The meridional dimensions (deltas) of the islands and gaps are then shrunk to infinitesimal values while more islands are added. The result is a porous ridge that allows a limited volume of fluid to leak through. Details of the topographic features such as shown in Figure 1 generally are unresolvable (and often unknown), and these uncertainties limit the predictive capabilities of course-grid models. Limits on predictability can be assessed by allowing the dimensions of the above islands to vary randomly and computing ensembles of solutions from the porous medium theory.

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